

# NONLINEAR EFFECTS OF POWER AMPLIFICATION ON MULTICARRIER SPREAD SPECTRUM SYSTEMS\*

Vuk Borich, Je-Hong Jong, Jack East, Wayne E. Stark

The University of Michigan  
Department of Electrical Engineering and Computer Science  
Ann Arbor, Michigan

## Abstract

We present a study of the distortion caused by power amplifiers driven by multicarrier spread-spectrum (MCSS) signals. The details of a time-domain technique for nonlinear amplifier characterization are presented as an alternative to the conventional, network analyzer methods. The bandpass nonlinear model and the associated AM-AM and AM-PM transfer characteristics are employed in order to predict the spectral regrowth as a function of the spreading gain. The predictions differ by approximately 2 dB from the measured results.

## 1. Introduction

Efficient power amplification is an essential consideration in the overall performance of today's wireless communication systems. The amplifiers used in such systems often operate near the 1-dB compression point in order to maximize power efficiency. Operating near the 1-dB compression point causes signal distortion in the case of varying-envelope signals.

An important measure of the nonlinear distortion is the spectral regrowth, as quantified by the so-called adjacent channel power ratio. The simplest of models used for spectral regrowth prediction is the so-called Memoryless, Bandpass Nonlinear Model. The model is appealing because of its simplicity. In this paper, we apply this simple model to the case of MCSS systems, which have raised a considerable interest due to their resistance to multipath fading interference.

## 2. Memoryless, Bandpass Nonlinear Model

In the bandpass nonlinear model, the instantaneous voltage at the output of a nonlinear device is represented as a series expansion of the input as:

$$y[x(t)] = \sum_{n=0}^{\infty} c_n x^n(t) \quad (1)$$

\*This work is supported by the Army Research Office under grant DAAH04-96-1-0001

If the input is a narrowband-modulated signal of the form  $x(t) = a(t) \cos(2\pi f_c t + \phi(t))$ , the output will generally consist of distortion products centered at integer multiples of the carrier frequency. In the majority of cases, however, we are only interested in the in-band products (those centered around the center frequency). By the assumption of the *bandpass* nonlinearity, the remaining frequency content is either removed by an ideal bandpass filter at the output, or it naturally falls outside of the system bandwidth.

If the assumed form of the input signal is substituted in (1), and if only the fundamental component of the output is retained, the output is of the form [1]:

$$y_o(t) = f[a(t)] \cos(2\pi f_c t + \phi(t)), \text{ where} \\ f(a[t]) = \sum_{n=1, 3, \dots} c_n a^n(t) \frac{n!}{2^{n-1} \cdot n-1! \cdot n+1!} \quad (2)$$

The function  $f[\cdot]$  represents the so-called AM-AM distortion, and is traditionally determined by single-tone measurements performed on a network analyzer.

The single-tone measurements also reveal that the output phase is a function of the input amplitude. Although this effect is not a consequence of the proposed form of the instantaneous output in equation (1), it is assumed that the output can be expressed as

$$y_o[t] = f[a(t)] \cos(2\pi f_c t + \phi(t) + \theta[a(t)]), \quad (3)$$

where  $\theta[a(t)]$  is the so-called AM-PM characteristic.

The coefficients of a polynomial fit to eq. (2) can be used in order to extract  $c_{2i-1}$ , the odd-order coefficients of the expansion in eq. (1). This is usually not of interest in spectral regrowth simulations, since we are only concerned with the signals in a narrow band around the carrier frequency. Equation (3), therefore, represents the bandpass nonlinear model which is the basis of the work presented in this paper.

## 3. Time-Domain Amplifier Characterization

Conventionally, the AM-AM and AM-PM characteristics are obtained using single-tone amplitude sweeps on a network analyzer. Recently, Heutmaker et.

al. [2] have pointed out the advantages of using digitally-modulated signals in order to obtain the transfer curves. Time and bandwidth (data-rate) dependencies of the nonlinear characteristics exhibited by some amplifiers [2] are obscured by the conventional measurement techniques, but are quite easily observed with the alternative, time-domain techniques. In [2], a digitally-modulated signal was used to obtain the transfer characteristics. Presently, there are no results which would indicate the optimum modulation scheme to be employed in the measurements. We have adopted the double-sideband, suppressed carrier (DSB-SC) format. In part, the scheme was chosen for its inherent simplicity; another reason for using DSB-SC modulation is the fact that it is identical to the familiar two-tone intermodulation testing.

The basic schematic of the measurement method is shown in figure 1:

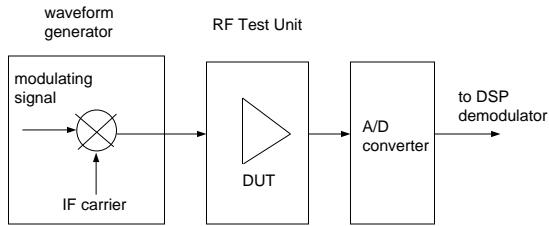


Figure 1: Basic Measurement Schematic

The test signal is generated on an arbitrary waveform synthesizer, and applied to the amplifier. The output is sampled and stored in an A/D converter. The stored output is digitally processed in order to obtain the AM-AM and AM-PM characteristics.

Commercial waveform synthesizers usually operate in the megahertz range, and are therefore not suitable for testing at microwave frequencies. To overcome this limitation, an “RF Test Unit” was devised. The schematic of this circuit is shown in figure 2.

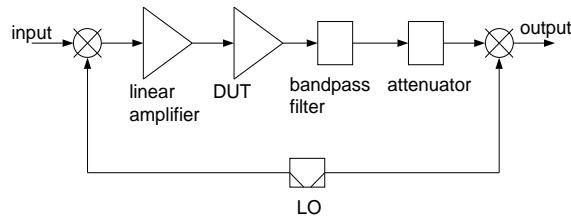


Figure 2: The “RF Test Unit”

The test signal is first upconverted to a suitable microwave frequency. The signal is kept at a low level in order to prevent unwanted distortion by the mixer-upconverter. Because of the low input signal level, it is usually necessary to add a low-distortion amplifier so that the saturation characteristics of the amplifier can be

observed. Following the amplifier is a bandpass filter centered at the carrier frequency. Finally, an attenuator is used in order to minimize the distortion of the mixer-downconverter.

As mentioned before, the test signal is a DSB-SC-modulated sinusoid. The modulating signal is a low frequency sine wave. The output of the signal generator may be represented as:

$$x(t) = A \sin(2\pi f_S t) \sin(2\pi f_{IF} t) \quad (4)$$

The low frequency sinusoid simulates the power sweep on a network analyzer. However, the signal in equation (4) is fundamentally different than an amplitude sweep, because it consists of two tones separated by  $2f_S$  and centered at  $f_{IF}$ . In other words, this signal has a non-zero bandwidth and may therefore be used to study bandwidth (data-rate) dependence of the nonlinearity. The amplifier tested in this paper showed no significant frequency dependence and hence such effects are not modeled here.

In order to extract the amplitude and phase modulation components introduced by the nonlinearity, we note that the signal presented to the A/D converter in figure 1 is of the form:

$$y(t) = f[A \sin(2\pi f_S t)] \sin(2\pi f_{IF} t + \phi[A \sin(2\pi f_S t)]) \quad (5)$$

The signal in eq. (5) is an amplitude- and phase-modulated sinusoid. The functions  $f[ \cdot ]$  and  $\phi[ \cdot ]$  are the desired AM-AM and AM-PM characteristics.

A simple method to extract the transfer curves would be to use coherent detection; with this method, eq. (5) is multiplied by  $\sin(2\pi f_{IF} t)$  and  $\cos(2\pi f_{IF} t)$ , and the products are lowpass-filtered. It can easily be shown that  $f[ \cdot ]$  and  $\phi[ \cdot ]$  can be extracted from the filtered outputs through simple mathematical manipulations. A potential difficulty with this approach is the delay associated with the A/D triggering circuitry. If the delay is significant (as is the case when the carrier frequency approaches one half of the A/D sampling rate) the transmitter and receiver carriers are out of phase and the detection error may become significant.

To avoid potential problems associated with coherent detection, the Hilbert transform approach is adopted in this paper [3]. With this approach, the analytic representation of the received signal is formed as:

$$z(t) = y(t) + j\tilde{y}(t),$$

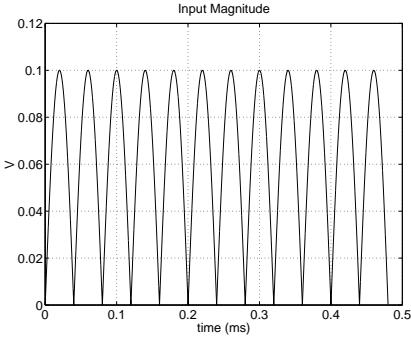
where  $\tilde{y}(t)$  is the Hilbert transform of  $y(t)$ . For the case of the signal in eq. (5) it can then be shown that:

$$|f[A \sin(2\pi f_S t)]| = \sqrt{y^2(t) + \tilde{y}^2(t)}, \text{ and}$$

$$\phi[A \sin(2\pi f_S t)] =$$

$$\begin{cases} \text{atan} \frac{\tilde{y}}{y} - 2\pi f_{IF} t & f[A \sin(2\pi f_S t)] > 0 \\ \text{atan} \frac{\tilde{y}}{y} - 2\pi f_{IF} t - \pi & f[A \sin(2\pi f_S t)] < 0 \end{cases}$$

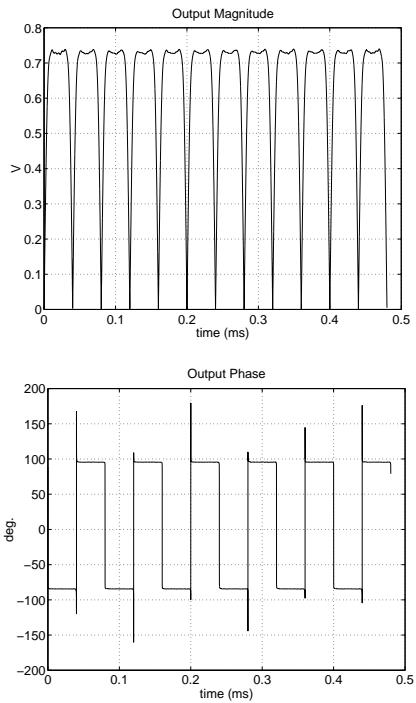
The modulating signal frequency used in the measurements was 12.5 kHz, and the IF carrier frequency was 400 kHz. The amplitude of the test signal (eq. 4) is shown in figure 3:



**Figure 3: Magnitude of Input Envelope**

The phase is not shown due to space limitations, but it simply alternates between -90 and +90 degrees as seen from eq. (4).

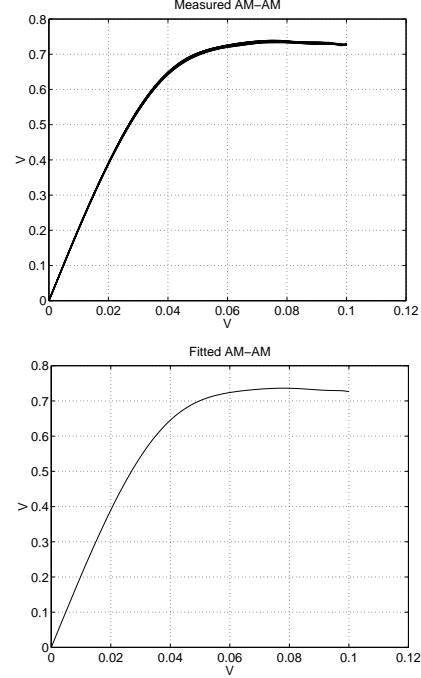
The input signal was applied to the amplifier as described above. The output is shown in figure 4:



**Figure 4: Amplitude and Phase of Output Envelope**

As seen in figure 4, the amplifier exhibited negligible

phase distortion. For that reason, the AM-PM characteristic was excluded from the analysis. A plot of the output vs. input amplitude characteristic yields the desired AM-AM curve. The measured data and the corresponding polynomial fit are shown in figure 5:

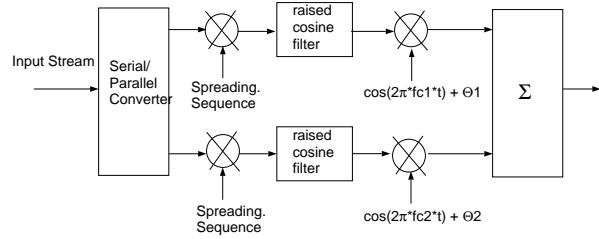


**Figure 5: Measured and Fitted AM-AM Data**

The fitted AM-AM data is used in the spectral regrowth calculations to be described in section 5.

#### 4. Transmitter Model

The model of the MCSS transmitter is shown in figure 6.



**Figure 6: Transmitter Model**

The multicarrier system studied in this paper employs two carriers. The input sequence is divided in two by the serial to parallel converter. Each stream is then spread by a random spreading sequence and filtered by a raised-cosine filter; the raised-cosine filter parameter was set at 0.5. The filtered data streams are multiplied

by the carriers whose phases are randomly selected. The carrier frequencies are set so that the spectra of the two substreams are adjacent to each other. The frequency of one of the carriers ( $f_{c1}$ ) was set to zero in this simple, two-carrier case. The sum of the two sequences then constitutes the MCSS signal used in our work.

Spreading gain is defined here as the ratio of the bit duration of the incoming data stream and the spreading sequence. We later vary the spreading gain in order to quantify its effects on the spectral spreading.

The input data stream consists of 256 bits, at a data rate of 61.035 kHz. The power spectral density (normalized to 0 dBm/Hz) of the input signal with a spreading gain of 2 is shown in figure 7.

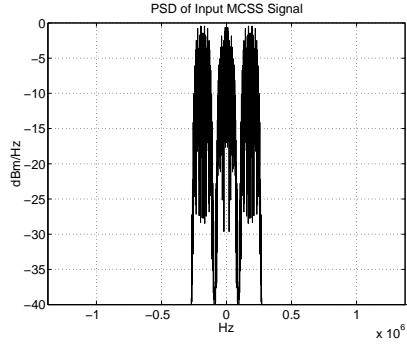


Figure 7: PSD of Input Signal

## 5. Results

The model of section 1 is particularly easy to apply in this case, where AM-PM distortion is negligible. Assuming that  $x(t)$  is the input MCSS waveform, the output of the bandpass filter in figure 2 is

$$f[x(t)]\cos(2\pi f_c t),$$

so that the input of the A/D converter is just  $f[x(t)]$ . Function  $f[*]$  was determined in section 3, and it is readily applied to calculate the output. The calculated output spectrum is shown in figure 8.

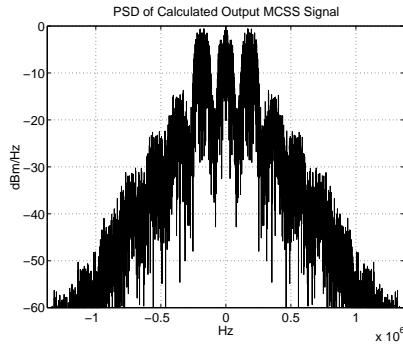


Figure 8: Predicted Output PSD

The adjacent channel power ratio (ACPR) is defined as the ratio of the adjacent- and in-channel

power. If the FFT of the output signal is denoted as  $Y(k)$ , then the in-channel and adjacent-channel power (in dBW) are given by:

$$P_{\text{in-channel}} = 10\log \frac{2}{N} \sum_{k=1}^{\frac{B}{2fres}} Y^2(k), \text{ and}$$

$$P_{\text{adj-channel}} = 10\log \frac{1}{N} \sum_{k=\frac{B}{2fres}+1}^{\frac{3B}{2fres}} Y^2(k),$$

$$k = \frac{B}{2fres} + 1$$

where  $B$  is the channel bandwidth,  $N$  is the FFT length, and  $fres$  is the frequency resolution.

The difference in the calculated and predicted ACPR is shown in figure 9:

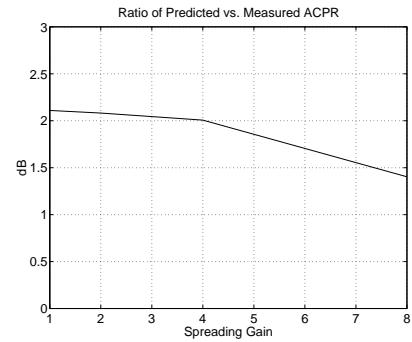


Figure 9: Error in ACPR

## 6. Conclusions

The memoryless, bandpass nonlinear model was applied to the case of Multicarrier Spread Spectrum Signals in order to predict the spectral regrowth. The disagreement between the measured and predicted ACPR was approximately 2 dB. The details of an alternative, time-domain method for determining the AM-AM and AM-PM characteristics have been presented, and the method was successfully applied in the measurements and simulation.

## References

- [1] S. Sabaroff, "Relationship between rms and instantaneous characteristics of nonlinear devices," *Proc. IEEE*, pp. 1967, December 1967.
- [2] M. Heutmaker, J. Welch and E. Wu, "Using Digital Modulation to Measure and Model RF Amplifier Distortion," *Applied Microwaves & Wireless*, pp. 34, March/April 1997.
- [3] E.E. Azzouz and A.K. Nandi, *Automatic Modulation Recognition of Communication Signals*, Kluwer Academic Publishers, Norwell, MA 1996.